AN EVALUATION OF METHODS

FOR EXTENDING STUB SURVIVOR CURVES OF

PHYSICAL PROPERTY

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ABSTRACT

The purpose of this investigation was to evaluate the effectiveness of the Gompertz-Makeham Formula, Iowa Type Curves, and the Retirement Rate Method for extending stub survivor curves of physical property. It was also desired to investigate the possible effects of length of the available stub, general shape of the survivor curve, and number of units in the group upon the effectiveness of the chosen methods.

Data for six property groups was selected in such a manner as to give wide variation in general shape of the survivor curve, type of property, average service life, and size of the group. Two hypothetical stubs were created from each set of data. One stub represented the data available when 70 per cent of the property still survived, while the other represented the situation when only 40 per cent survived. Each of the three methods was applied to each of the twelve stubs to obtain a complete survivor curve which best fitted the available stub. In a few cases, the data was such that a particular method was unable to produce any survivor curve to extend the stub.

Effectiveness of the methods was evaluated in terms of the accurracy of prediction of average service life and goodness of fit of the extension with the actual data. The error of the prediction of average life was represented by a percentage of actual average life. For comparison purposes, goodness of fit was represented by the sum of the squared

deviations between the extension and the actual data, divided by the number of observations. Effectiveness in regard to average service life was relatively consistent with effectiveness in regard to goodness of fit.

The results of the study may be summarized as follows. The results with the Gompertz-Makeham Formula were quite unsatisfactory. The Retirement Rate Method gave results which were not unreasonable for the short stubs and very accurate results for the long stubs. It was found that the manner of applying the Iowa Type curves was such that the additional points available in the long stub were of little value in choosing a complete survivor curve. Therefore, the Iowa Method generally gave the same results with the short and long stubs. The Iowa Method results were quite satisfactory. The effectiveness of these latter two methods was apparently unaffected by the general shape of the survivor curve or the size of the property group. The quantity of data afforded by a greater average service life appears to improve the results.

These findings indicate the use of the Retirement Rate Method when a very accurate extension of relatively long stub is required, the use of the Iowa Type Curves for extending short stubs, and the use of Iowa Type Curves for reasonable approximations to longer stubs with a minimum of effort.

CHAPTER I

INTRODUCTION

Object of the investigation.--Survivor curves of physical property portray the retirement pattern of a group of property units from the time of acquiring the group until the last unit has been retired. The fraction (in per cent) of the group surviving is plotted against age. The group may be an actual one--"original group"--or a hypothetical typical group. We can begin plotting this curve year by year for an actual group, but until the last unit is retired we will not have a complete curve. These incomplete survivor curves are called stubs. They may also result in some cases of attempting to portray typical groups. Without a complete survivor curve we cannot compute the average life for the units, nor do we have a picture of the retirement pattern of the units retired in the later years.

Suppose a group of 100 units is acquired. Let us say that after five years 30 of these units have been retired from service. We have plotted the stub survivor curve for the five years. If it is possible to chose a complete curve to correspond to this stub we will have prediction of the pattern of retirement and the average life for the 70 units still surviving. The method used to chose a complete curve, or to extend the stub, will obviously effect the accuracy of our prediction. At present several methods are advocated with equal assurances as to pre-eminence. This accuracy will also be influenced by the size of the stub available and possibly other factors. This investigation is designed primarily to compare three methods of extension and to study the effect of length of stub, while some inferences may be obtained in regard to other variables. The method used is to create stubs from actual, complete data, and then compare the extension with the actual data.

Obviously, in predicting the retirement of physical property many factors in addition to historical data must be considered. For example, it would be foolish to extend a stub curve at the point when the plant changed from two to three shift operation. However, this does not mean that historical data is of no value in estimating property life. It is certainly used in most cases, if only on the basis of subjective experience. Statistical analysis of service life data puts this experience in accurate, significant form. The accurate predictions obtained in this investigation using certain methods of extending stub survivor curves indicates the validity of considering only the historical factor for some cases. In extending a stub, the historical data for the retired portion of the group is being used to predict for the surviving portion.

Statistical Analysis of Service Life Data.--The expected service life of a unit or group of industrial property becomes an important consideration at times of acquiring, valuating, or depreciating the property. The statistical analysis of historical data concerning the service lives of a group of identical or similar units will provide an important factor in the estimation of expected service life. The weight of this factor--in relation to expected usage, managerial plans, possibility of technical innovation--will depend on the particular property and circumstances.

The statistical analysis of the data will yield a figure for average service life of past units and in many cases (depending on the data), durves showing the pattern of retirement experienced in the past.

Many of the statistical methods used in the analysis of industrial property retirements have developed from the analysis of human mortality. Although the term "industrial property" is frequently used, by far the major part of studies to date have been restricted to utility property. Lack of satisfactory data has been one reason for little application in other fields. Present emphasis on the depreciation problem--especially in relation to Internal Revenue allowances and taxation--has heightened the interest in evaluation, development, and wider application of this type of analysis. The picture of retirement dispersion which is produced provides information for depreciation methods other than straightline. Study of the various survivor curves may show the effect of changes in the equipment, usage, or other external influences, e.g., war, depression, etc.

There are two broad classes of statistical approaches to the analysis of service life data. These are actuarial and turnover methods. Only the actuarial methods will be discussed here, since only they produce survivor curves and retirement dispersion curves. The turnover methods (1, 2) have the advantage of requiring less data, however, only the figure for average life may be obtained.

The three principal actuarial methods of analysis are the annual rate method, the original group method, and the individual unit method. The annual rate method--the best but requiring the most data--will be described first. Although usually applied to a band of years, it will first be considered as applied to one year. Data must be available for the number of units, classified by ages, which are in service at the beginning of the year. The number retired during the year, and their respective ages, must be known.[1] From this data, the retirement rate for each age group may be computed (number retired/number exposed to retirement in the age group). By subtracting these rates (expressed as decimals) from one, the survivor rates are found. These survivor rates for the year are applied successively to the per cent surviving at the beginning of the year for each age group. This process produces the retirement pattern which would exist if these retirement rates would persist year after year for the respective ages. If the derived percentages surviving are plotted against age, a survivor curve such as that in Figure 1 will be produced, provided the original curve is smoothed and includes an age where the survivors are zero. A fuller description of this method will be found in Grant and Norton's "Depreciation" (3).

The same approach may be applied to a band of years. The figures used are merely the sum of the number exposed in each age group and the number retired in each age group. Smoother and more reliable results can be obtained. If the per cent retired is plotted for each age interval, the retirement frequency dispersion curve is obtained (Figure 1). The average service life is obtained through some process of integration, usually a numerical approximation from total years of service delivered, divided by the number of units. A curve of retirement rate versus age may also be plotted.

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[1]The assumption is made that all units are placed or retired in the middle of a year.

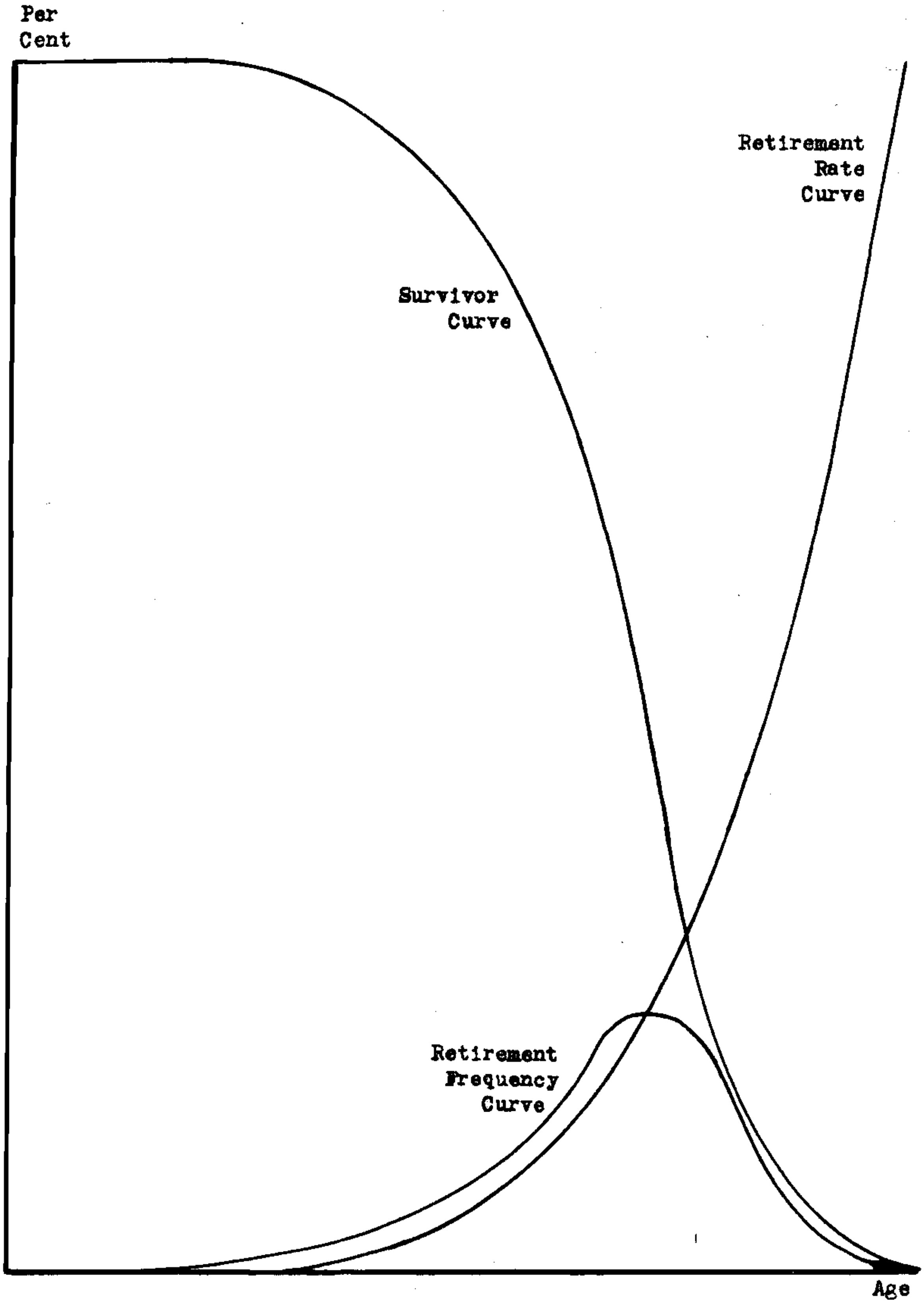

Figure 1. Survivor Curve--Retirement Frequency Curve--
Retirement Rate Curve

The data represented by the curves in Figure 1 may be plotted as number surviving (or retired) versus age (or interval) or per cent surviving versus age (as shown) or per cent surviving versus per cent of average age. This last process gives what are called the generalized survivor and frequency[2] curves. The advantage of eliminating consideration of the particular numbers or ages involved is apparent from the standpoint of basic study.

The second of the actuarial methods is the original group method. Data needed includes the ages of retired and surviving members of the group, all of which were placed in the same year. Survivor and frequency curves may be plotted directly. The curves will be complete if all members of the group have been retired. The composite original group method merely sums the data for several original groups. The multiple original group method utilizes the per cent surviving at a given date for a number of original groups of varying ages. The per cent surviving in each group represents a point on the survivor curve (4).

The last actuarial method to be discussed is the individual unit method (5). The only data needed is the ages at retirement of a large number of units. All the units considered are construed to be the 100 per cent group and the per cent surviving at each age may be readily plotted. For young, growing, or declining accounts, the results may be misleading.

In connection with the determination of survivor curves by actuarial methods, one more important aspect merits discussion. Workers at the Iowa State College Engineering Experiment Station, notably Robley

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[2]Ten per cent intervals of average service life are usually used for plotting the retirement frequency curve.

Winfrey and Edwin Kurtz, through a process of smoothing, classifying, and generalizing the generalized survivor curves of numerous groups of property (eventually 176), finally arrived at 18 type survivor curves (6). The curves are differentiated primarily by the position of the mode in relation to the average life and the height of the mode. These curves are supposed to be typical of the survivor curves of all types of industrial property and are so spaced over the possible range that it is postulated that one may be chosen to closely approximate any survivor curve encountered in practice. They may be used to extend stub curves, and tabulated data corresponding to each type curve may be used in connection with a similar actual curve. Other groups of typical or type curves exist, e.g., those of Patterson (7).

The question of the value of statistical interpretations of historical data when estimating the probable service life of a unit or group is too broad and of too little relevance to be discussed in detail here. However, it might be said in summary of the controversy that this type analysis produces only one factor necessary for the estimate of service life. Judgment must always be used in regard to the future position of the property in the business, and future prospects in the industry. Perhaps it is fair to say that among the proponents of the viewpoint and in their industry, too much emphasis has been placed on this tool, while probably other industrial areas have overlooked the value to be derived.

A few references to advocates and detractors of the theory may be of interest to the reader. Some of the pros and cons are well summarized by Grant and Norton (8). Benson gives a defense of the theory (9). Nash, although the originator of a turnover method which bears his name, strongly emphasizes the limitations of the approach (10). Logan and Dwight (11) ably show the necessity of judgment and consider some of the factors involved in a service life estimate.

Stub Survivor Curves.--Unless an annual rate study is made at such a time as all members of some age group have been retired, only part of a survivor curve will result. This incomplete survivor curve is called a stub. In the case of the original group method, a stub will result unless all members have been retired. By the nature of the individual unit method, a complete curve results, but this is the key to the weakness of the method. For fairly long-lived property, the stub curves are probably a more common occurrence than complete curves. Also, the extension of a stub curve gives estimates for the remaining units of the group, while a complete survivor curve means that all members of the group are already retired. Extension of stub curve, therefore, provides a direct method of prediction for the group, while complete survivor curves only provide data which is more or less applicable to other groups.

The methods of extending stub curves may be classified in two ways--by the type of method used or by which curve is first extended. The problem first exists as to whether to smooth the stub. This is usually done by eye, although least squares or other mathematical means may be employed. The first general method of extending stubs is by eye and judgment. The method is usually applied to the survivor stub. This method has the obvious advantages of simplicity and speed. However, although the quality of the extension will vary with the experience of the worker, accurate and consistent results cannot be expected.

Numerous mathematical approaches to the problem have been used. Probably the most widely used method has been the Gompertz-Makeham Formula (12, 13), originally developed for the study of human mortality. The formula used is derived by considering the combined effect of deterioration through age and random accident. This method is applied to the survivor stub. This approach, with some modifications, has been used extensively by Bell Telephone. The depreciation committee of the National Association of Railroad and Utilities Commissioners recommends extension of the retirement rate (or ratio) curve (14). This is done by the method of least squares or by a method of R. A. Fisher. Pearson or Gram-Charlier distribution curves may be used in many cases for smoothing, but apparently for extension only where the data is very nearly complete. The use of truncated normal distribution curves as advocated by Kimball (15) has as yet received little attention. This method is only applicable to nearly complete data (about 20 per cent surviving).

The third major method of extending stub curves is through choosing a closely fitting type curve. In the case of the Iowa curves, each of the 18 generalized curves is plotted for several values of average life on tracing paper. By visual comparison with the various types and average service lives, the closest fitting curve is chosen. Winfrey had done some preliminary work on the effect of length of stub on validity of matching (16) and gives subjective comments on the goodness of fit of his type of curves (17). Patterson type curves or curves developed by the particular company might be used in a similar manner.

Criteria of evaluating extension.--From a review of the literature several criteria of the effectiveness of an extension appear. The first obvious criterion of a method is its relative ease of application. The computation involved in some of the mathematical methods may become quite laborious. In practice, this must be weighed against the scope of the original data and the purpose of the analysis.

A second criterion is the attainment of the proper shape for the curve. A good result for average service life may be obtained only because several errors over the length of the curve balance each other. To retain the great advantage of the actuarial approach in giving a picture of the retirement pattern, the best predictable shape must result. The problem arises here of dependence of latter parts of the curve upon earlier parts. Of course, the whole principle of extension rests upon a firm belief in this dependence. Little prediction of the exact shape of a survivor curve can be made from the situation to be portrayed, although in some cases, it seems the company may have more effect upon the shape than the type of property.

A good extension should give a reasonably accurate value for the average service life. This is the most important single figure to be derived from the statistical study of service life data.

A fourth criterion of the effectiveness of a method of extension is the amount of stub required for a valid extension. Obviously, with a 90 per cent stub available, extension by eye will probably produce a rather good result. Ability to extend shorter stubs with accuracy certainly adds to the value of a method.

In the course of methematically formulating the 18 Iowa Type Curves, Winfrey found (18) that certain methods gave good results for some types of curves, while the results were quite poor with other types. Since the general type of the curve is not known at the time of extension, this weakness could be quite serious.

During the investigation, two other factors arose which were of interest in regard to their relation to effectiveness of extension. One was the number of units in the property group. Obviously, a small group will give data with rather abrupt changes in per cent surviving. A second factor which probably effects the effectiveness of extension is the ratio of the average service life to the time interval used--one year in almost all cases. This influences the number of points available when extending the data for a given per cent surviving.

CHAPTER II

GENERAL PROCEDURE

The first step was to select the methods of extension to be used for the tests and select or gather survival data on property groups. It was decided to use data published by the Iowa State Engineering Experiment Station since data of this sort is usually held in confidence by the companies involved, and also it was felt that the Iowa data would be fully as satisfactory as data gathered particularly for this study.

The general procedure used was to take from each set of complete data figures representing a stub at a time when 70 per cent of the group still survived, and figures representing a stub with 40 per cent surviving. These particular levels were chosen so as to see the effect of length of stub at what is probably the shortest "reasonable" stub and at a point past where 50 per cent of the group is retired. Extremely short stubs cannot be expected to give satisfactory extensions. Extremely long stubs may easily be extended comparatively accurately by eye. The points of demarcation for 70 and 40 in the data were made as close as possible to these round figures (the exact points are shown in the tabulated data).

It was felt best to do no preliminary smoothing. The mathematical methods, of course, are actually smoothing and then extending. Any subjective preliminary smoothing of "unreasonable" values would destroy the objectivity of the experiment.

Thus, for each group of data two stubs were used. Each method was applied to each stub. The results of these extensions were then compared to the actual--complete--data in regard to average service life and goodness of fit.

A graphical example of the results of one application is shown in Figure 2. The points of the original data are marked to denote which comprise the 70 per cent stub and the 40 per cent stub. The solid curves represent the results of extending these stubs by the Retirement Rate Method. The data is Property Group 60--37 manure spreaders. The data gives an average life of 10.9 years, extension of the short stub gives 10.2, and extension of the long stub gives 11.0. The goodness of fit is substantially better for the long stub extension. Description of the method, the data, tabulated results, and other information concerning this example will be found in the appropriate subsequent sections.

<u>Selection of Methods</u>.--The Gompertz-Makeham Formula, the Iowa Type Curves, and extension of the retirement rate curve with a polynomial were chosen as the methods to be used. These are probably the three most widely used methods and represent three different approaches to the problem--mathematical extension of the survivor curve, graphical choice of a type survivor curve, and mathematical extension of the retirement rate curve. The Gompertz-Makeham Formula was used in its usual form, since this is the one most frequently suggested and recommended, and since the Bell Telephone modification was unavailable. The Iowa Type Curves were chosen since they are probably the most widely publicized, and also the necessary information was available. The Retirement Rate Method was chosen as a

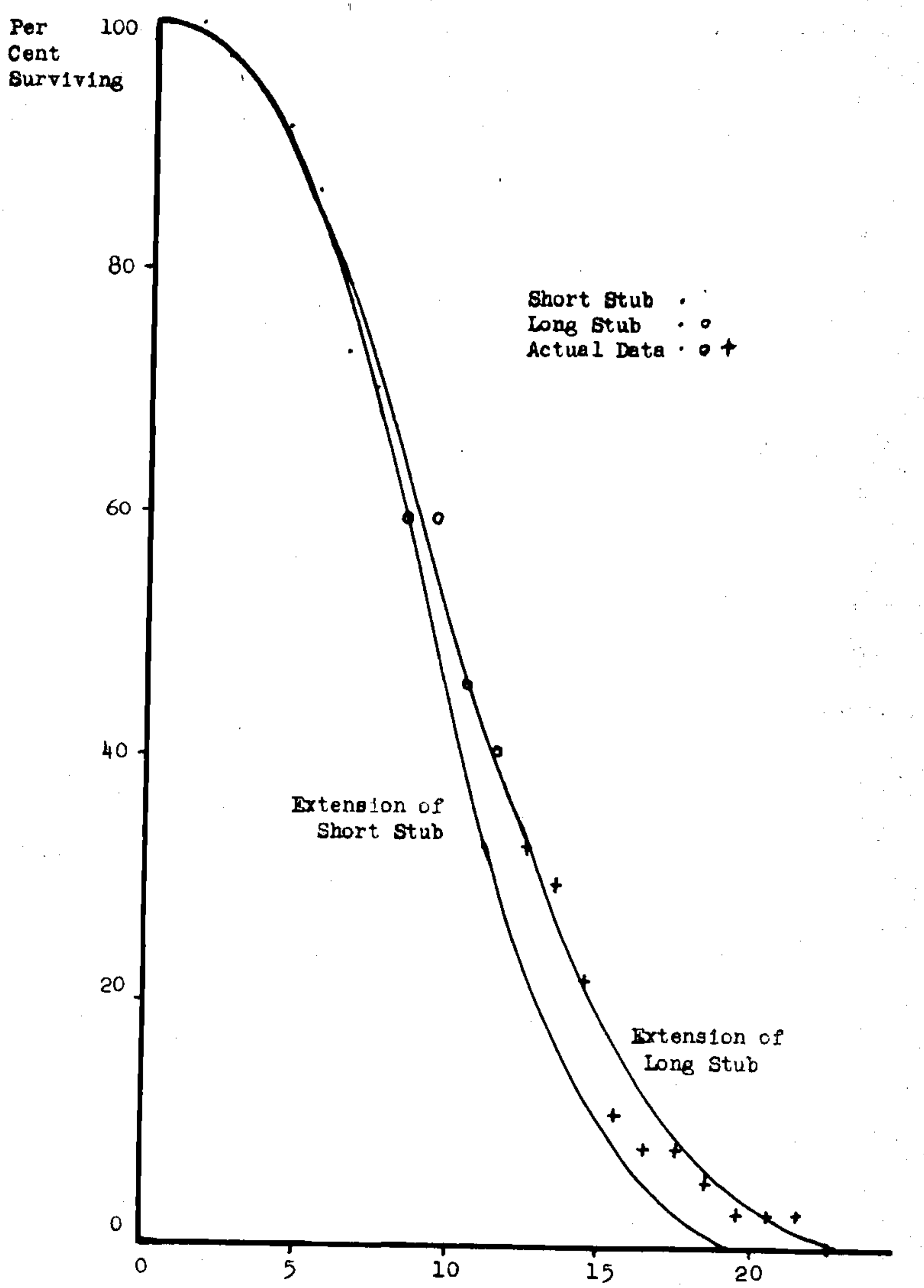

Figure 2. Retirement Rate Method Applied to Group 60

different approach to the problem and in the light of the claims made for it by the National Association of Railroad and Utilities Commissioners (19).

Some study of the literature was devoted to the possibility of extending stub frequency curves. It was decided that this was not feasible. First, there is considerable difference of opinion as to what is the appropriate type of frequency curve--e.g., Pearson Type I or III, Gram-Charlier, truncated normal, etc. Second, if the application were attempted assumptions or approximations concerning type, average life, maximum life, etc., would be such as to produce no objective validity in the extension, except that present in the assumptions. That is, most of the answer to the problem would have to be assumed before the method was applied and this would be reflected in the results given by the method.

Selection of data.--In selecting the property groups for study, it was desired that there be a reasonable representation of the variables considered relevant to the effectiveness of extension--shape, average service life, number of units in the group. Also, an attempt was made to get a sample of different kinds of property to avoid possible bias from this source.

The data on 65 property groups presented in Iowa Bulletin 103 (20) was the population from which the data was selected. Although 111 additional groups were used in Bulletin 125 (21), no original data is given. It was decided to select data for six property groups.

From Iowa Bulletin 125 (22) the following tabulation was made of the final Iowa classification of the 176 groups of data. The capital letters with subscripts refer to the Iowa Type Curve which best fitted the data. "S" denotes a symmetrical retirement frequency curve, "R" denotes a right modal curve, and "L" a left modal. Larger subscripts signify more peaked retirement frequency curves.

S_0	5	L_0	8	R_1	5
S_1	13	L_1	15	R_2	5
S_2	11	L_2	17	R_3	19
S_3	7	L_3	15	R_4	24
S_4	10	L_4	8	R_5	7
S_5	3	L_5	3		
S_6	1				
	50		66		60
					Total 176

In view of this distribution of shapes in the 176 groups, it was decided to select two each of symmetrical, left modal, and right modal groups. S_1, S_4, L_1, L_3, R_3, and R_4 were chosen as the classes from which the six sets of data would be chosen. It will be noticed that where the frequency of occurance of two shapes is about the same, shapes with the greatest difference were selected. Choice from the several sets of data having the desired shape was achieved through a subjective attempt at variability in kind of property, average life, and number of units in the group.

Description and Tabulation of Data.--The following descriptions of the six property groups are taken directly from Iowa Bulletin 103 (23). The quotation is verbatum except for the omission of the second part of the Iowa code number--referring to the number of the group in a set gathered from the same source. The code numbers used here will be used throughout the thesis in referring to the six groups of data.

9. The life experience of central office equipment used in seven common battery offices of the New Jersey Division of the New York Telephone Co. The equipment was valued at $848,109.

29. The life experience of about 75, 40-watt, Mazda incandescent lamps. The data for these tables were taken in January, 1915.

33. The experience of 781 steam locomotives on the U.P.R.R., C.B. and Q.R.R., and C.R.I. and P.Ry. as compiled by E. J. Kates, engineer on the staff of the Nebraska State Railway Commission, Nov. 10, 1910.

43. The life experience of 26,146 railway cross ties of Douglas fir species when treated with zinc chloride. They were set in 1900 in various places on the Southern Pacific System and subjected to heavy traffic. Data were collected by the Forest Products Laboratory, Madison, Wis.

54. The life experience of 1,107 box cars of 28,000 pounds capacity, which were installed 1869 to 1880. The data were obtained from protestant's exhibit No. 71, before the Interstate Commerce Commission, Valuation Docket 327, Great Northern Railway Company and Montana Eastern Railway Company, 1923.

60. The life experience of 37 manure spreaders of 45 to 80 bushels capacity which were used in Hardin County, Iowa, 1890 to 1924. They were used 5 to 55 days a year. Data compiled by the Iowa Engineering Experiment Station, 1924.

It might be of interest at this point to list the Iowa type curves as to shape and average service life which were found (24) to best fit the complete data.

9	$8.7L_1$	43	$9.3S_4$
29	$12.3R_4$	54	$28.2R_3$
33	$25.4L_3$	60	$10.5S_1$

All the data is the result of the individual unit type of analysis. In the tabulated data that follows, only the first two columns are taken directly from Bulletin 103 (25). The third column is found by subtracting the value in the second column from the previous value in the third column. The fourth column is determined by dividing a value in the second column by the corresponding value in the third column. The two horizontal lines drawn through the data denote the stubs which were extended--the first line at about 70 per cent surviving and the second at about 40 per cent surviving.

Table 1. Actual Data for Property Group 9

Central Office Equipment Valued at $848,109

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
0.0- .5	0.0	100.0	0.0
0.5- 1.5	3.8	100.00	3.8
1.5- 2.5	6.0	96.2	6.2
2.5- 3.5	6.7	90.2	7.4
3.5- 4.5	5.3	83.5	6.3
4.5- 5.5	17.4	78.2	22.3
5.5- 6.5	10.4	60.8	17.1
6.5- 7.5	4.5	50.4	8.9
7.5- 8.5	3.0	45.9	6.5
8.5- 9.5	6.0	42.9	14.0
9.5-10.5	2.7	36.9	7.3
10.5-11.5	1.9	34.2	5.6
11.5-12.5	6.2	32.3	19.2
12.5-13.5	3.1	26.1	11.9
13.5-14.5	4.4	23.0	19.1
14.5-15.5	3.8	18.6	20.4
15.5-16.5	5.5	14.8	37.2
16.5-17.5	0.6	9.3	6.5
17.5-18.5	1.2	8.7	13.8
18.5-19.5	3.0	7.5	40.0
19.5-20.5	2.5	4.5	55.6
20.5-21.5	0.1	2.0	5.0
21.5-22.5	0.7	1.9	36.8
22.5-23.5	0.1	1.2	8.3
23.5-24.5	0.1	1.1	9.1
24.5-25.5	1.0	1.0	100.0
25.5-26.5	0.0	0.0	

Table 2. Actual Data for Property Group 29

75 Incandescent Lamps

Age Interval in 100 hours	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
0.0- 0.5	0.0	100.0	0.0
0.5- 1.5	0.0	100.0	0.0
1.5- 2.5	0.0	100.0	0.0
2.5- 3.5	0.0	100.0	0.0
3.5- 4.5	0.0	100.0	0.0
4.5- 5.5	0.0	100.0	0.0
5.5- 6.5	9.0	100.0	9.0
6.5- 7.5	1.5	91.0	1.6
7.5- 8.5	2.5	89.5	2.8
8.5- 9.5	11.0	87.0	12.6
9.5-10.5	7.0	76.0	9.2
10.5-11.5	10.5	69.0	15.2
11.5-12.5	24.5	58.5	14.9
12.5-13.5	20.0	34.0	58.8
13.5-14.5	7.0	14.0	50.0
14.5-15.5	4.0	7.0	57.1
15.5-16.5	3.0	3.0	100.0
16.5-17.5	0.0	0.0	

Table 3. Actual Data for Property Group 33

781 Steam Locomotives

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
0.0- 0.5	0.0	100.0	0.0
0.5- 1.5	0.0	100.0	0.0
1.5- 2.5	0.0	100.0	0.0
2.5- 3.5	0.0	100.0	0.0
3.5- 4.5	0.0	100.0	0.0
4.5- 5.5	0.0	100.0	0.0
5.5- 6.5	0.0	100.0	0.0
6.5- 7.5	0.0	100.0	0.0
7.5- 8.5	0.0	100.0	0.0
8.5- 9.5	0.5	100.0	0.5
9.5-10.5	0.5	99.5	0.5
10.5-11.5	1.0	99.0	1.0
11.5-12.5	1.0	98.0	1.0
12.5-13.5	1.5	97.0	1.5
13.5-14.5	1.5	95.5	1.6
14.5-15.5	3.0	94.0	3.2
15.5-16.5	3.0	91.0	3.3
16.5-17.5	4.0	88.0	4.5
17.5-18.5	4.0	84.0	4.8
18.5-19.5	5.0	80.0	6.2
19.5-20.5	5.0	75.0	6.7
20.5-21.5	5.5	70.0	7.9
21.5-22.5	4.5	64.5	7.0
22.5-23.5	5.2	60.0	8.7
23.5-24.5	4.3	54.8	7.8
24.5-25.5	4.5	50.5	8.9
25.5-26.5	4.8	46.0	10.4
26.5-27.5	4.2	41.2	10.2
27.5-28.5	4.0	37.0	10.8
28.5-29.5	3.8	33.0	11.5
29.5-30.5	3.5	29.2	12.0
30.5-31.5	3.2	25.7	12.5
31.5-32.5	3.3	22.5	14.7
32.5-33.5	2.7	19.2	14.1
33.5-34.5	2.6	16.5	15.8

Table 3. Continued

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
34.5-35.5	2.1	13.9	15.1
35.5-36.5	2.2	11.8	18.6
36.5-37.5	1.6	9.6	16.7
37.5-38.5	1.5	8.0	18.8
38.5-39.5	1.5	6.5	23.1
39.5-40.5	1.0	5.0	20.0
40.5-41.5	1.0	4.0	25.0
41.5-42.5	0.8	3.0	26.7
42.5-43.5	0.6	2.2	27.3
43.5-44.5	0.6	1.6	37.5
44.5-45.5	0.3	1.0	30.0
45.5-46.5	0.3	0.7	42.9
46.5-47.5	0.3	0.4	75.0
47.5-48.5	0.02	0.1	20.0
48.5-49.5	0.06	0.08	75.5
49.5-50.5	0.02	0.02	100.0
50.5-51.5	0.00	0.00	

Table 4. Actual Data for Property Group 43

26,146 Railway Cross Ties

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
0.0- 0.5	0.0	100.0	0.0
0.5- 1.5	0.0	100.0	0.0
1.5- 2.5	0.0	100.0	0.0
2.5- 3.5	0.0	100.0	0.0
3.5- 4.5	0.0	100.0	0.0
4.5- 5.5	0.0	100.0	0.0
5.5- 6.5	4.4	100.0	4.4
6.5- 7.5	2.2	95.6	2.3
7.5- 8.5	28.5	93.4	30.5
8.5- 9.5	21.4	64.9	33.0
9.5-10.5	17.2	43.5	39.5
10.5-11.5	22.4	26.3	85.2
11.5-12.5	0.2	3.9	5.1
12.5-13.5	3.7	3.7	100.0
13.5-14.5	0.0	0.0	

Table 5. Actual Data for Property Group 54

1,107 Box Cars

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
0.0- 0.5	0.0	100.0	0.0
0.5- 1.5	0.7	100.0	0.7
1.5- 2.5	1.7	99.3	1.7
2.5- 3.5	0.3	97.6	0.3
3.5- 4.5	0.9	97.3	0.9
4.5- 5.5	0.5	96.4	0.5
5.5- 6.5	0.2	95.9	0.2
6.5- 7.5	0.6	95.7	0.6
7.5- 8.5	1.2	95.1	1.3
8.5- 9.5	0.6	93.9	0.6
9.5-10.5	1.0	93.3	1.1
10.5-11.5	1.0	92.3	1.1
11.5-12.5	0.8	91.3	0.9
12.5-13.5	0.6	90.5	0.7
13.5-14.5	1.7	89.9	1.9
14.5-15.5	1.3	88.2	1.5
15.5-16.5	2.9	86.9	3.3
16.5-17.5	1.4	84.0	1.7
17.5-18.5	2.4	82.6	2.9
18.5-19.5	1.7	80.2	2.1
19.5-20.5	0.3	78.5	0.4
20.5-21.5	1.2	78.2	1.5
21.5-22.5	1.9	77.0	2.5
22.5-23.5	0.9	75.1	1.2
23.5-24.5	2.1	74.2	2.8
24.5-25.5	2.2	72.1	3.1
25.5-26.5	3.5	69.9	5.0
26.5-27.5	3.4	66.4	5.1
27.5-28.5	3.4	63.0	5.4
28.5-29.5	5.0	59.6	8.4
29.5-30.5	7.1	54.6	13.0
30.5-31.5	4.1	47.5	8.6
31.5-32.5	4.8	43.4	11.1
32.5-33.5	5.0	38.6	13.0
33.5-34.5	4.8	33.6	14.3

Table 5. Continued

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
34.5-35.5	5.5	28.8	19.1
35.5-36.5	3.7	23.3	15.9
36.5-37.5	4.3	19.6	21.9
37.5-38.5	4.6	15.3	30.1
38.5-39.5	1.9	10.7	17.8
39.5-40.5	2.1	8.8	23.9
40.5-41.5	0.6	6.7	9.0
41.5-42.5	1.4	6.1	23.0
42.5-43.5	0.7	4.7	14.9
43.5-44.5	1.3	4.0	32.5
44.5-45.5	0.7	2.7	25.9
45.5-46.5	0.7	2.0	35.0
46.5-47.5	0.9	1.3	69.2
47.5-48.5	0.29	0.4	72.5
48.5-49.5	0.00	0.11	0.0
49.5-50.5	0.11	0.11	100.0
50.5-51.5	0.00	0.00	

Table 6. Actual Data for Property Group 60

37 Manure Spreaders

Age Interval in years	Per cent Retired in interval	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent
0.0- 0.5	0.0	100.0	0.0
0.5- 1.5	0.0	100.0	0.0
1.5- 2.5	2.7	100.0	2.7
2.5- 3.5	2.7	97.3	2.8
3.5- 4.5	2.7	94.6	2.9
4.5- 5.5	5.4	91.9	5.9
5.5- 6.5	13.5	86.5	15.6
6.5- 7.5	2.7	73.0	3.7
7.5- 8.5	10.8	70.3	15.4
8.5- 9.5	0.0	59.5	0.0
9.5-10.5	13.6	59.5	22.9
10.5-11.5	5.4	45.9	11.8
11.5-12.5	8.1	40.5	20.0
12.5-13.5	2.7	32.5	8.3
13.5-14.5	8.1	29.7	27.3
14.5-15.5	10.8	21.8	50.0
15.5-16.5	2.7	10.8	25.0
16.5-17.5	0.0	8.1	0.0
17.5-18.5	2.7	8.1	33.3
18.5-19.5	2.7	5.4	50.0
19.5-20.5	0.0	2.7	0.0
20.5-21.5	0.0	2.7	0.0
21.5-22.5	2.7	2.7	100.0
22.5-23.5	0.0	0.0	

CHAPTER III

GOMPERTZ-MAKEHAM FORMULA

Procedure.--The procedure used in applying the Gompertz-Makeham formula to a stub survivor curve will be referred to only briefly here. However, a fuller description of the method and an illustrative example are to be found in Iowa Bulletin 125 (26).

The Gompertz-Makeham formula is expressed as follows:

$$l_x = ks^x g^{c^x}$$

where l_x equals the number living at age x, and k, s, g, and c are constants. The formula is derived as a mathematical expression combining two forces causing retirement--chance and deterioration. The equation put in terms for solution by taking its logarithm, then breaking the sum of the logarithms from x to $x + 4t - 1$ into four equations. By differencing both sides of these four equations twice, five more equations result.

In the actual arithmetic, a value is chosen for x, the starting point of age, and t, the number of observations which will be in each of the four groups of data. The number of points available--best not including more than one at 100 per cent--divided by four and used as a whole number gives a value for t. By using the most recent observations, if possible, and counting back, x is established. We now have four sets of survival figures with t figures in each set and the figures begin at age x. We take the sum of the logarithms for each set and difference these sums. Then these sums are differenced. The resulting figures enable c to be found. Then g, s, and k are successively found. Using the logarithms of these four constants and the appropriate values for x, l_x may be found for the required ages.

Summary of Results.--In the first application to the 12 stubs, the most recent points were included. In only three cases was the application successful. The reason for failure in the other cases was that the figures for the first or second difference of the sums of the logarithms of the four sets were not greater for greater ages. This required taking the logarithm of a negative number and c could not be found. Geometrically, this means that the slope of the data was not changing fast enough--or changing in the wrong direction--so that the formula could not be fitted.

For the unsuccessful cases, x was made one less. This drops the most recent point and picks up one earlier point. This sometimes gave success. Otherwise, various other values of x were tried and t was made smaller. This process was halted when the possible combinations were exhausted or when it was felt that so many recent values had been discarded as to make the extension fruitless. Seven of the twelve stubs were fitted. The values used and tried for x and t will be found in Table 7.

The fitting of stub 33-40[3] failed for a reason different from that

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[3]This notation will be continued throughout the thesis. The first figure indicates the property group, the second, the per cent surviving of the stub.

given above. The differences in the logarithms were small, however, c was found. Then, each successive constant determined was farther from a reasonable value. Apparently, the data so narrowly fitted the formula that the constants became extreme and quite sensitive. A tendency in this direction may be seen in the results of Table 8 where k for 9-40 has become very large, where the logarithm of g for 29-70 and 60-70 has become very small, and where c for 29-70 is very large.

Table 7. Values of x and t Used in Gompertz-Makeham Fitting

Stub	Successful x	Successful t	Unsuccessful x	Unsuccessful t
9-70			1.5	1
			.5	1
9-40	.5	2	1.5	2
29-70	7.5	1	8.5	1
29-40	6.5	2		
33-70	9.5	3		
33-40			11.5	4
			10.5	4
			9.5	4
			8.5	4
			15.5	3
			14.5	3
43-70			5.5	1
43-40			6.5	1
54-70	.5	6	2.5	6
			1.5	6
54-40	1.5	8		
60-70	3.5	1	4.5	1
60-40			4.5	2
			3.5	2
			2.5	2
			8.5	1

Table 8. Constants Obtained in the Application of Gompertz-Makeham

Stub	c	g	s	k
9-40	1.0965	.0896	1.229	1127
29-70	9.4124	1.0000[4]	.9854	101.7
29-40	1.543	.997	1.005	98.00
33-70	1.102	.8484	1.041	102.6
54-70	1.0737	.9315	1.002	108.0
54-40	1.1326	.9868	.9963	100.7
60-70	3.6923	1.0000[5]	.9813	101.5

[4] $\log g = -3.52 \times 10^{-12}$

[5] $\log g = -1.854 \times 10^{-5}$

Tabulated Results.--Tables 9 through 13 give the per cent surviving values for the fitted Gompertz-Makeham curves for the seven stubs where fitting was successful. Illogical values such as those over one hundred, or those which rise and then fall have not been eliminated. A comparison of these tabulated values with the actual data may prove interesting. Comparison in terms of average life and goodness of fit will be found in the Conclusions.

Table 9. Gompertz-Makeham Result with Stub 9-40

Central Office Equipment Valued at $848,109

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	101.0	100.0
0.5	100.0	100.0
1.5	96.3	96.2
2.5	90.6	90.2
3.5	83.2	83.5
4.5	74.0	78.2
5.5	64.0	60.8
6.5	53.5	50.4
7.5	43.1	45.9
8.5	33.3	42.9
9.5	24.6	36.9
10.5	17.3	34.2
11.5	11.5	32.3
12.5	7.2	26.1
13.5	4.3	23.0
14.5	2.3	18.6
15.5	1.2	14.8
16.5	0.6	9.3
17.5	0.2	8.7
18.5	0.1	7.5
19.5	0.0	4.5
20.5		2.0
21.5		1.9
22.5		1.2
23.5		1.1
24.5		1.0
25.5		0.0

Table 10. Gompertz-Makeham Results with Stubs 29-70 and 29-40

75 Incandescent Lamps

Age in 100 hours	29-70 Per cent Surviving	29-40 Per cent Surviving	Actual Per cent Surviving
0.0	101.7	97.7	100.0
0.5	100.9	97.9	100.0
1.5	99.4	98.0	100.0
2.5	98.0	98.4	100.0
3.5	96.5	98.4	100.0
4.5	95.1	98.2	100.0
5.5	93.7	97.6	100.0
6.5	92.4	96.4	91.0
7.5	91.0	94.3	89.5
8.5	89.5	91.0	87.0
9.5	87.1	85.7	76.0
10.5	76.0	78.0	69.0
11.5	23.9	67.3	58.5
12.5	0.0	53.4	34.0
13.5		37.2	14.0
14.5		21.3	7.0
15.5		9.0	3.0
16.5		2.4	0.0
17.5		0.3	
18.5		0.0	

Table 11. Gompertz-Makeham Results with Stub 33-70

781 Steam Locomotives

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	87.0	100.0
0.5	88.0	100.0
1.5	90.0	100.0
2.5	91.9	100.0
3.5	93.6	100.0
4.5	95.2	100.0
5.5	96.5	100.0
6.5	97.6	100.0
7.5	98.4	100.0
8.5	98.9	100.0
9.5	99.1	99.5
10.5	98.9	99.0
11.5	98.3	98.0
12.5	97.2	97.0
13.5	95.6	95.5
14.5	93.5	94.0
15.5	90.8	91.0
16.5	87.7	88.0
17.5	83.9	84.0
18.5	79.7	80.0
19.5	75.0	75.0
20.5	69.8	70.0
21.5	64.3	64.5
22.5	58.4	60.0
23.5	52.4	54.8
24.5	46.3	50.5
25.5	40.2	46.0
26.5	34.3	41.2
27.5	28.6	37.0
28.5	23.4	33.0
29.5	18.6	29.2
30.5	14.5	25.7
31.5	10.9	22.5
32.5	7.9	19.2
33.5	5.5	16.5

Table 11. Continued

Age in years	Per cent Surviving	Actual Per cent Surviving
34.5	3.8	13.9
35.5	2.4	11.8
36.5	1.5	9.6
37.5	0.9	8.0
38.5	0.4	6.5
39.5	0.2	5.0
40.5	0.1	4.0
41.5	0.1	3.0
42.5	0.0	2.2
43.5		1.6
44.5		1.0
45.5		0.7
46.5		0.4
47.5		0.1
48.5		0.08
49.5		0.02
50.5		0.00

Table 12. Gompertz-Makeham Results with Stubs 54-70 and 54.40

1,107 Box Cars

Age in years	54-70 Per cent Surviving	54-40 Per cent Surviving	Actual Per cent Surviving
0.0	99.2	99.4	100.0
0.5	98.9	99.2	100.0
1.5	98.5	98.6	99.3
2.5	98.1	98.0	97.6
3.5	97.6	97.4	97.3
4.5	97.0	96.7	96.4
5.5	96.4	96.1	95.9
6.5	95.7	95.4	95.7
7.5	95.0	94.7	95.1
8.5	94.2	93.9	93.9
9.5	93.3	93.1	93.3
10.5	92.4	92.2	92.3
11.5	91.4	91.2	91.3
12.5	90.3	90.3	90.5
13.5	89.1	89.2	89.9
14.5	87.9	88.0	88.2
15.5	86.5	86.8	86.9
16.5	85.1	85.4	84.0
17.5	83.6	83.9	82.6
18.5	82.0	82.3	80.2
19.5	80.3	80.6	78.5
20.5	78.5	78.7	78.2
21.5	76.5	76.6	77.0
22.5	74.5	74.4	75.1
23.5	72.4	72.0	74.2
24.5	70.1	69.4	72.1
25.5	67.9	66.6	69.9
26.5	65.4	63.6	66.4
27.5	62.9	60.4	63.0
28.5	60.3	57.0	59.6
29.5	57.7	53.4	54.6
30.5	54.9	49.7	47.5
31.5	52.1	45.7	43.4
32.5	49.2	41.4	38.6

Table 12. Continued

Age in years	54-70 Per cent Surviving	54-40 Per cent Surviving	Actual Per cent Surviving
33.5	46.3	37.5	33.6
34.5	42.7	33.3	28.8
35.5	40.4	29.2	23.3
36.5	37.4	25.1	19.6
37.5	34.8	21.2	15.3
38.5	31.6	17.5	10.7
39.5	28.7	14.1	8.8
40.5	25.9	11.0	6.7
41.5	23.2	8.3	6.1
42.5	20.7	6.1	4.7
43.5	18.2	4.3	4.0
44.5	15.9	2.9	2.7
45.5	13.7	1.8	2.0
46.5	11.7	1.1	1.3
47.5	9.9	0.6	0.4
48.5	8.2	0.3	0.11
49.5	6.8	0.2	0.11
50.5	5.5	0.1	0.00
51.5	4.4	0.0	
52.5	3.4		
53.5	2.7		
54.5	2.0		
55.5	1.5		
56.5	1.1		
57.5	0.8		
58.5	0.5		
59.5	0.4		
60.5	0.2		
61.5	0.1		
62.5	0.1		
63.5	0.1		
64.5	0.0		

Table 13. Gompertz-Makeham Results with Stub 60-70

37 Manure Spreaders

Age in years	Per cent Surviving	Actual per cent Surviving
0.0	101.5	100.0
0.5	100.5	100.0
1.5	98.6	100.0
2.5	96.7	97.3
3.5	94.6	94.6
4.5	91.8	91.9
5.5	86.5	86.5
6.5	72.9	73.0
7.5	40.9	70.3
8.5	5.1	59.5
9.5	0.0	59.5
10.5		45.9
11.5		40.5
12.5		32.5
13.5		29.7
14.5		21.8
15.5		10.8
16.5		8.1
17.5		8.1
18.5		5.4
19.5		2.7
20.5		2.7
21.5		2.7
22.5		0.0

CHAPTER IV

IOWA TYPE CURVES

Procedure.--Detailed data for the 18 Iowa type curves and recommendations for their use in extending stubs will be found in Iowa Bulletin 125. Briefly, it is recommended that a set of graphs be prepared for each type curve plotted at several average lives. The data for the stub is plotted on the same scale, and by visual comparison the proper type curve and average life are selected. Then data for the extension may be found through interpolation from tabular data published for each type curve.

The process described above, using a set of graphs with each type curve plotted at several values of average life, would be quite satisfactory if numerous stubs were to be extended. However, when it is desired to extend only a few, the labor in preparing the tool would be quite excessive. The method used here requires more work for each fitting but eliminates the preparation of the numerous preliminary graphs and should give equally satisfactory results.

Bulletin 125 (27) gives data and graphs for the 18 type curves in generalized form (per cent average life on the horizontal axis). By examining the stub data, a reasonable value for average life may be estimated. For example, if the stub data shows seventy per cent surviving at an age of 8.5 years and the survivors are decreasing by about 12 per cent per year, the figure 11 years might be chosen as a first approximation of average life. By assuming this value, and one or two higher and lower, the stub data may be put into generalized form with respect to each of the assumed average lives. In comparing these to the generalized 18 type curves, it soon becomes evident whether larger or smaller values of average life will be more profitable. This method of approximating the average service life, plotting the generalized stubs corresponding to these assumed values, and visually comparing these results with the generalized type curves, results in subjective determination of the best fitting Iowa type for the stub, as well as a refined estimate of average life.[6]

Summary of Results.--The Iowa type curve and average life which was judged to best fit each of the 12 stubs is given in Table 14. It will be noted that in all cases the average life is the same for the longer and shorter stubs. In all but one case, the type curve is the same. This is due to the fact that the generalized type survivor curves are well differentiated at the upper end (and at the lower end), but come close together and cross each other in the middle range. This means that the additional points of the longer stub--in the middle range--were frequently of little or no value in selecting the proper type curve.

Generally, there was more difficulty in selecting the proper modality than the proper peakedness, e.g., S_3, R_3, L_3 might all fit fairly well. In a few cases, the problem arose of fairly good fits with different type curves when the average life was changed, e.g., apparently,

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[6]Bulletin 125 recommends that average life only be estimated to the nearest year.

equally good fits with $9L_0$ and $8S_1$. This made it impossible to firmly select first the average life or type and then select the other. However, in most cases, the average life could be chosen with reasonable certainty first, and then the type selected.

Table 14. Average Lives and Iowa Types Used to Extend Stubs

Stub	Average Life	Iowa Type
9-70	8	L_0
9-40	8	L_0
29-70	13	S_3
29-40	13	R_3
33-70	25	L_3
33-40	25	L_3
43-70	9	R_5
43-40	9	R_5
54-70	28	R_2
54-40	28	R_2
60-70	10	S_1
60-40	10	S_1

Note: All ages given in years, except Group 29 which is in 100 hour units.

Tabulated Results.--

Table 15. Iowa Method Results for Stubs 9-70 and 9-40

Central Office Equipment Valued at $848,109

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	100.0	100.0
0.5	98.5	100.0
1.5	92.5	96.2
2.5	86.4	90.2
3.5	78.5	83.5
4.5	71.0	78.2
5.5	62.9	60.8
6.5	55.7	50.4
7.5	48.2	45.9
8.5	41.7	42.9
9.5	34.9	36.9
10.5	29.3	34.2
11.5	23.7	32.3
12.5	19.4	26.1
13.5	15.2	23.0
14.5	11.9	18.6
15.5	8.9	14.8
16.5	6.4	9.3
17.5	5.0	8.7
18.5	3.4	7.5
19.5	2.3	4.5
20.5	1.6	2.0
21.5	0.9	1.9
22.5	0.6	1.2
23.5	0.3	1.1
24.5	0.2	1.0
25.5	0.1	0.0
26.5	0.0	

Table 16. Iowa Method Results for Stubs 29-70 and 29-40

75 Incandescent Lamps

Age in 100 hours	29-70 Per cent Surviving	29-40 Per cent Surviving	Actual Per cent Surviving
0.0	100.0	100.0	100.0
0.5	100.0	100.0	100.0
1.5	100.0	99.8	100.0
2.5	100.0	99.5	100.0
3.5	99.9	98.8	100.0
4.5	99.8	97.9	100.0
5.5	99.4	96.5	100.0
6.5	98.2	94.4	91.0
7.5	95.7	91.4	89.5
8.5	92.1	88.2	87.0
9.5	85.8	83.3	76.0
10.5	77.3	77.0	69.0
11.5	68.1	71.1	58.5
12.5	53.1	60.3	34.0
13.5	43.8	48.9	14.0
14.5	31.9	36.6	7.0
15.5	22.7	26.1	3.0
16.5	13.3	15.0	0.0
17.5	7.8	8.5	
18.5	4.3	4.5	
19.5	1.8	1.5	
20.5	0.6	0.2	
21.5	0.2	0.0	
22.5	0.0		

Table 17. Iowa Method Results for Stubs 33-70 and 33-40

781 Steam Locomotives

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	100.0	100.0
0.5	100.0	100.0
1.5	100.0	100.0
2.5	100.0	100.0
3.5	100.0	100.0
4.5	99.9	100.0
5.5	99.8	100.0
6.5	99.7	100.0
7.5	99.4	100.0
8.5	99.1	100.0
9.5	98.5	99.5
10.5	97.8	99.0
11.5	96.9	98.0
12.5	95.8	97.0
13.5	94.3	95.5
14.5	92.3	94.0
15.5	89.8	91.0
16.5	86.7	88.0
17.5	83.0	84.0
18.5	78.5	80.0
19.5	73.5	75.0
20.5	68.2	70.0
21.5	62.6	64.5
22.5	57.0	60.0
23.5	51.5	54.8
24.5	46.4	50.5
25.5	41.5	46.0
26.5	37.0	41.2
27.5	33.0	37.0
28.5	29.4	33.0
29.5	26.2	29.2
30.5	23.2	25.7
31.5	20.6	22.5
32.5	18.2	19.2
33.5	16.0	16.5

Table 17. Continued

Age in years	Per cent Surviving	Actual Per cent Surviving
34.5	14.1	13.9
35.5	12.3	11.8
36.5	9.6	9.6
37.5	9.0	8.0
38.5	7.7	6.5
39.5	6.5	5.0
40.5	5.4	4.0
41.5	4.4	3.0
42.5	3.5	2.2
43.5	2.8	1.6
44.5	2.2	1.0
45.5	1.7	0.7
46.5	1.3	0.4
47.5	0.9	0.1
48.5	0.7	0.08
49.5	0.5	0.00
50.5	0.3	
51.5	0.2	
52.5	0.1	
53.5	0.1	
54.5	0.0	

Table 18. Iowa Method Results for Stubs 43-70 and 43-40

26,146 Railway Cross Ties

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	100.0	100.0
0.5	100.0	100.0
1.5	100.0	100.0
2.5	100.0	100.0
3.5	100.0	100.0
4.5	100.0	100.0
5.5	99.4	100.0
6.5	96.7	95.6
7.5	88.7	93.4
8.5	67.5	64.9
9.5	35.4	43.5
10.5	9.2	26.3
11.5	0.8	3.9
12.5	0.0	3.4
13.5		0.0

Table 19. Iowa Method Results for Stubs 54-70 and 54-40

1,107 Box Cars

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	100.0	100.0
0.5	99.8	100.0
1.5	99.5	99.3
2.5	99.0	97.6
3.5	98.5	97.3
4.5	98.0	96.4
5.5	97.3	95.9
6.5	96.8	95.7
7.5	95.9	95.1
8.5	95.3	93.9
9.5	94.2	93.3
10.5	93.1	92.3
11.5	92.2	91.3
12.5	90.9	90.5
13.5	89.8	89.9
14.5	88.2	88.2
15.5	86.9	86.9
16.5	84.8	84.0
17.5	83.5	82.6
18.5	81.3	80.2
19.5	79.0	78.5
20.5	77.0	78.2
21.5	74.3	77.0
22.5	72.1	75.1
23.5	68.9	74.2
24.5	65.5	72.1
25.5	62.9	69.9
26.5	59.1	66.4
27.5	56.2	63.0
28.5	52.1	59.6
29.5	49.0	54.6
30.5	44.6	47.5
31.5	41.4	43.4
32.5	37.0	38.6
33.5	32.6	33.6

Table 19. Continued

Age in years	Per cent Surviving	Actual Per cent Surviving
34.5	29.4	28.8
35.5	25.3	23.3
36.5	22.3	19.6
37.5	18.6	15.3
38.5	15.2	10.7
39.5	12.9	8.8
40.5	10.9	6.7
41.5	8.4	6.1
42.5	6.4	4.7
43.5	5.0	4.0
44.5	3.6	2.7
45.5	2.4	2.0
46.5	1.6	1.3
47.5	0.8	0.4
48.5	0.5	0.11
49.5	0.2	0.11
50.5	0.1	0.00
51.5	0.0	

Table 20. Iowa Method Results for Stubs 60-70 and 60-40

37 Manure Spreaders

Age in years	Per cent Surviving	Actual Per cent Surviving
0.0	100.0	100.0
0.5	100.0	100.0
1.5	99.5	100.0
2.5	98.1	97.3
3.5	95.5	94.6
4.5	91.4	91.9
5.5	86.1	86.5
6.5	79.5	73.0
7.5	71.8	70.3
8.5	63.4	59.5
9.5	54.5	59.5
10.5	45.5	45.9
11.5	36.6	40.5
12.5	28.2	32.4
13.5	20.5	29.7
14.5	13.9	21.8
15.5	8.6	10.8
16.5	4.6	8.1
17.5	1.9	8.1
18.5	0.5	5.4
19.5	0.0	2.7
20.5		2.7
21.5		2.7
22.5		0.0

CHAPTER V

RETIREMENT RATE CURVES

Procedure.--The extension of stub survivor curves through the method of extending the derived retirement rate curve and then translating this back into retirement frequencies and survival figures is advocated by the depreciation committee of the National Association of Railroad and Utilities Commissioners (28) and mentioned in many other sources. Theoretically, the curve may be represented by a polynomial, in x of first, second, third, or higher order. However, it would seem that in most cases the second order is most desirable for extension. A typical retirement rate curve is shown in Figure 1. Winfrey, in Iowa Bulletin 125 (29) comments on a "step" found in the retirement rate curve for left modal types. The step is of varying importance in various members of the left modal class. This would indicate a better fit with a third order equation, but the stubs used here are not of sufficient length to give adequate data concerning the step--its beginning and ending points--and therefore attempt to consider it would not prove profitable.

The retirement rate stubs were extended by an equation of the type $y = a + bx + cx^2$ where y equals the retirement rate, x equals the period, and a, b, and c are constants. The fitting and extension were accomplished by a method proposed by R. A. Fisher (30). The method is such that the ordinates are fitted and extended directly, and determination of the constants involves considerable additional labor. Since these constants would be of little value, they were not found. The initial point for the first half year period is dropped before extension, so that all the periods will be equal.

The arithmetical procedure of the method described by Fisher for finding the values of the best fitting second order polynomial may be briefly described as follows. The data is tabulated in the order of increasing x. Five columns should be available to the left of the data. In the column next to the data, figures representing the successive sums of the observations are entered. The next column tabulates the successive sums of the second column. The totals of these first three columns are used to find three constants. These are in turn used to find three other constants. These three determine the terminal values for the three left hand columns (see Table 21). The fifth column is built up from its terminal value (ΔY_1) by successively adding the terminal value ($\Delta^2 Y_1$) of the sixth column. The fourth column, the desired polynomial values, is built up from its terminal value (Y_1) by successively adding the terms from the fifth column. The series may be built up for higher values of x by merely building the fifth and fourth columns in the opposite direction.

Summary of Results.--Table 21 gives the values used for building up the required columns of successive differences. Their meaning and use are explained more fully in Fisher (31).

The extension was successful in all but one case. The data for stub 9-40 was definitely decreasing in the later values. This caused the fitting to produce a curve with its maximum before the end of the stub (concave downward)--a situation that would obviously give unreasonable results.

Tabulated Results.--In Tables 22 through 27 the fitted retirement rates as well as the survival values are shown. The rates are shown as obtained. In determining the survival values, all negative rates and the values preceeding them were taken as zero. The series of rate values was extended until a figure of 100 per cent or more was obtained. Obviously a figure over 100 is illogical in retirement studies, but since we are merely extending a polynomial, such values will occur. No difficulty results, since 100 or more extinguishes the property group. In some cases, the property group was completely retired (to one decimal place) before 100 per cent retirement rate was reached, e.g., Groups 33 and 54.

Table 21. Terminal Differences for Retirement Rate Extensions[7]

Stub	Y_1	ΔY_1	$\Delta^2 Y_1$
9-70	20.18	-9.046	3.5570
9-40	10.07	2.006	-0.8374
29-70	16.34	-3.921	0.5143
29-40	49.38	-11.168	1.3081
33-70	7.78	-0.980	0.0641
33-40	11.19	-0.878	0.0348
43-70	33.66	-11.735	2.2528
43-40	42.61	-12.062	1.8825
54-70	3.40	-0.239	0.0114
54-40	11.46	-0.963	0.0431
60-70	13.18	-2.371	0.0728
60-40	17.86	-1.919	0.0728

[7]For meaning of these constants see Fisher (32).

Table 22. Retirement Rate Results for Stub 9-70

Central Office Equipment Valued at $848,109

Age Interval in years	Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	Actual Per cent Surviving at beginning of interval
0.0- 0.5		100.0	100.0
0.5- 1.5	5.3	100.0	100.0
1.5- 2.5	3.7	94.7	96.2
2.5- 3.5	5.6	91.2	90.2
3.5- 4.5	11.1	86.1	83.5
4.5- 5.5	20.2	76.5	78.2
5.5- 6.5	32.8	61.0	60.8
6.5- 7.5	48.9	41.0	50.4
7.5- 8.5	68.7	21.0	45.9
8.5- 9.5	91.9	6.6	42.9
9.5-10.5	118.8	0.5	36.9
10.5-11.5		0.0	34.2
11.5-12.5			32.3
12.5-13.5			26.1
13.5-14.5			23.0
14.5-15.5			18.6
15.5-16.5			14.8
16.5-17.5			9.3
17.5-18.5			8.7
18.5-19.5			7.5
19.5-20.5			4.5
20.5-21.5			2.0
21.5-22.5			1.9
22.5-23.5			1.2
23.5-24.5			1.1
24.5-25.5			1.0
25.5-26.5			0.0

Table 23. Retirement Rate Results for Stubs 29-70 and 29-40

75 Incandescent Lamps

Age Interval in 100 hours	29-70 Retirement Rate for interval in per cent	29-70 Per cent Surviving at beginning of interval	29-40 Retirement Rate for interval in per cent	29-40 Per cent Surviving at beginning of interval	Actual Per cent Surviving at beginning of interval
0.0- 0.5		100.0		100.0	100.0
0.5- 1.5	1.5	100.0	6.2	100.0	100.0
1.5- 2.5	0.3	100.0	1.7	100.0	100.0
2.5- 3.5	-0.4	100.0	-1.5	100.0	100.0
3.5- 4.5	-0.6	100.0	-3.4	100.0	100.0
4.5- 5.5	-0.3	100.0	-4.0	100.0	100.0
5.5- 6.5	0.5	100.0	-3.3	100.0	100.0
6.5- 7.5	1.9	99.5	-1.3	100.0	91.0
7.5- 8.5	3.7	97.6	2.0	100.0	89.5
8.5- 9.5	6.1	94.0	6.6	98.0	87.0
9.5-10.5	9.0	88.3	12.6	91.5	76.0
10.5-11.5	12.4	80.4	19.8	80.0	69.0
11.5-12.5	16.3	70.4	28.4	64.2	58.5
12.5-13.5	20.8	58.9	38.2	46.0	34.0
13.5-14.5	25.7	46.6	49.4	28.4	14.0
14.5-15.5	31.2	34.6	61.9	14.4	7.0
15.5-16.5	37.2	23.8	75.6	5.5	3.0
16.5-17.5	43.7	14.9	90.7	1.3	0.0
17.5-18.5	50.7	8.4	107.1	0.1	
18.5-19.5	58.2	4.1		0.0	
19.5-20.5	66.2	1.7			
20.5-21.5	74.8	0.6			
21.5-22.5	83.8	0.2			
22.5-23.5		0.0			

Table 24. Retirement Rate Results for Stubs 33-70 and 33-40

781 Steam Locomotives

Age Interval in years	33-70 Retirement Rate for interval in per cent	33-70 Per cent Surviving at beginning of interval	33-40 Retirement Rate for interval in per cent	33-40 Per cent Surviving at beginning of interval	Actual per cent Surviving at beginning of interval
0.0- 0.5		100.0		100.0	100.0
0.5- 1.5	0.4	100.0	-0.3	100.0	100.0
1.5- 2.5	0.1	100.0	-0.3	100.0	100.0
2.5- 3.5	0.0	100.0	-0.3	100.0	100.0
3.5- 4.5	-0.2	100.0	-0.2	100.0	100.0
4.5- 5.5	-0.2	100.0	-0.1	100.0	100.0
5.5- 6.5	-0.2	100.0	0.1	100.0	100.0
6.5- 7.5	-0.1	100.0	0.2	99.9	100.0
7.5- 8.5	0.0	100.0	0.5	99.7	100.0
8.5- 9.5	0.2	100.0	0.7	99.2	100.0
9.5-10.5	0.5	99.8	1.0	98.5	99.5
10.5-11.5	0.9	99.3	1.3	97.5	99.0
11.5-12.5	1.3	98.4	1.7	96.2	98.0
12.5-13.5	1.7	97.1	2.1	94.6	97.0
13.5-14.5	2.3	95.4	2.5	92.6	95.5
14.5-15.5	2.9	93.2	3.0	90.3	94.0
15.5-16.5	3.5	90.5	3.5	87.6	91.0
16.5-17.5	4.2	87.3	4.0	84.5	88.0
17.5-18.5	5.0	83.6	4.6	81.1	84.0
18.5-19.5	5.9	79.4	5.2	77.4	80.0
19.5-20.5	6.8	74.7	5.8	73.7	75.0
20.5-21.5	7.8	69.6	6.5	69.4	70.0
21.5-22.5	8.8	64.2	7.2	64.9	64.5
22.5-23.5	9.9	58.6	7.9	60.2	60.0
23.5-24.5	11.1	52.8	8.7	55.4	54.8
24.5-25.5	12.3	46.9	9.5	50.6	50.5
25.5-26.5	13.6	41.1	10.3	45.8	46.0
26.5-27.5	15.0	35.5	11.2	41.1	41.2
27.5-28.5	16.4	30.2	12.1	36.5	37.0
28.5-29.5	17.9	25.2	13.0	32.1	33.0
29.5-30.5	19.5	20.7	14.0	27.9	29.2
30.5-31.5	21.1	16.7	15.0	24.0	25.7
31.5-32.5	22.8	13.2	16.1	20.4	22.5
32.5-33.5	24.5	10.2	17.2	17.1	19.2
33.5-34.5	26.4	7.7	18.3	14.3	16.5

Table 24. Continued

Age Interval in years	33-70 Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	33-40 Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	Actual Per cent Surviving at beginning of interval
34.5-35.5	28.2	5.7	19.5	11.7	13.9
35.5-36.5	30.2	4.1	20.7	9.4	11.8
36.5-37.5	32.2	2.9	21.9	7.5	9.6
37.5-38.5	34.2	2.0	23.2	5.9	8.0
38.5-39.5	36.4	1.3	24.4	4.5	6.5
39.5-40.5	38.6	0.8	25.8	3.4	5.0
40.5-41.5	40.8	0.5	27.2	2.5	4.0
41.5-42.5	43.2	0.3	28.6	1.8	3.0
42.5-43.5	45.6	0.2	30.0	1.3	2.2
43.5-44.5	48.0	0.1	31.4	0.9	1.6
44.5-45.5	50.5	0.1	33.0	0.6	1.0
45.5-46.5		0.0	34.5	0.4	0.7
46.5-47.5			36.1	0.3	0.4
47.5-48.5			37.7	0.2	0.1
48.5-49.5			39.3	0.1	0.08
49.5-50.5			41.0	0.1	0.02
50.5-51.5				0.0	0.00

Table 25. Retirement Rate Results for Stubs 43-70 and 43-40

26,146 Railway Cross Ties

	43-70		43-40		Actual
Age Interval in years	Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	Per cent Surviving at beginning of interval
0.0- 0.5		100.0		100.0	100.0
0.5- 1.5	2.8	100.0	1.8	100.0	100.0
1.5- 2.5	-1.2	100.0	-1.2	100.0	100.0
2.5- 3.5	-3.0	100.0	-2.3	100.0	100.0
3.5- 4.5	-2.5	100.0	-1.5	100.0	100.0
4.5- 5.5	0.2	100.0	1.1	100.0	100.0
5.5- 6.5	5.2	99.8	5.7	98.9	100.0
6.5- 7.5	12.4	94.6	12.1	93.3	95.6
7.5- 8.5	21.9	82.9	20.4	82.0	93.4
8.5- 9.5	33.7	64.7	30.5	65.3	64.9
9.5-10.5	47.6	42.9	42.6	45.4	43.5
10.5-11.5	63.9	22.5	56.6	26.1	26.3
11.5-12.5	82.4	8.1	72.4	11.4	3.9
12.5-13.5	103.1	1.4	90.1	3.1	3.7
13.5-14.5		0.0	109.7	0.3	0.0
14.5-15.5				0.0	

Table 26. Retirement Rate Results for Stubs 54-70 and 54-40

1,107 Box Cars

Age Interval in years	54-70 Retirement Rate for interval in per cent	54-70 Per cent Surviving at beginning of interval	54-40 Retirement Rate for interval in per cent	54-40 Per cent Surviving at beginning of interval	Actual Per cent Surviving at beginning of interval
0.0- 0.5		100.0		100.0	100.0
0.5- 1.5	0.8	100.0	2.2	100.0	100.0
1.5- 2.5	0.8	99.2	1.9	97.8	99.3
2.5- 3.5	0.8	98.4	1.5	95.9	97.6
3.5- 4.5	0.8	97.6	1.2	94.5	97.3
4.5- 5.5	0.8	96.8	1.0	93.4	96.4
5.5- 6.5	0.8	96.0	0.7	92.5	95.9
6.5- 7.5	0.8	95.2	0.5	91.9	95.7
7.5- 8.5	0.8	94.4	0.4	91.4	95.1
8.5- 9.5	0.9	93.6	0.2	91.0	93.9
9.5-10.5	1.0	92.8	0.2	90.8	93.3
10.5-11.5	1.0	91.9	0.2	90.6	92.3
11.5-12.5	1.1	91.0	0.3	90.4	91.3
12.5-13.5	1.2	90.0	0.4	90.1	90.5
13.5-14.5	1.3	88.9	0.5	89.7	89.9
14.5-15.5	1.4	87.7	0.7	89.3	88.2
15.5-16.5	1.5	86.5	0.9	88.7	86.9
16.5-17.5	1.7	85.2	1.2	87.9	84.0
17.5-18.5	1.8	83.7	1.5	86.8	82.6
18.5-19.5	2.0	82.2	1.9	85.5	80.2
19.5-20.5	2.1	80.6	2.3	83.9	78.5
20.5-21.5	2.3	78.9	2.7	82.0	78.2
21.5-22.5	2.5	77.1	3.2	79.8	77.0
22.5-23.5	2.7	75.2	3.8	77.2	75.1
23.5-24.5	2.9	73.2	4.3	74.3	74.2
24.5-25.5	3.2	71.1	5.0	71.1	72.1
25.5-26.5	3.4	68.8	5.6	67.5	69.9
26.5-27.5	3.6	66.5	6.3	63.7	66.4
27.5-28.5	3.9	64.1	7.1	59.7	63.0
28.5-29.5	4.2	61.6	7.9	55.5	59.6
29.5-30.5	4.5	59.0	8.7	51.5	54.6
30.5-31.5	4.8	56.3	9.6	46.7	47.5
31.5-32.5	5.1	53.6	10.5	42.2	43.4

Table 26. Continued

Age Interval in years	54-70 Retirement Rate for interval in per cent	54-70 Per cent Surviving at beginning of interval	54-40 Retirement Rate for interval in per cent	54-40 Per cent Surviving at beginning of interval	Actual Per cent Surviving at beginning of interval
32.5-33.5	5.4	50.9	11.5	37.8	38.6
33.5-34.5	5.7	48.2	12.5	33.5	33.6
34.5-35.5	6.1	45.5	13.5	29.3	28.8
35.5-36.5	6.4	42.7	14.6	25.3	23.3
36.5-37.5	6.8	40.0	15.8	21.6	19.6
37.5-38.5	7.2	37.3	16.9	18.2	15.3
38.5-39.5	7.5	34.6	18.2	15.1	10.7
39.5-40.5	7.9	32.0	19.4	12.4	8.8
40.5-41.5	8.4	29.5	20.7	10.0	6.7
41.5-42.5	8.8	27.0	22.1	7.9	6.1
42.5-43.5	9.2	24.6	23.5	6.2	4.7
43.5-44.5	9.6	22.3	24.9	4.7	4.0
44.5-45.5	10.1	20.2	26.4	3.5	2.7
45.5-46.5	10.6	18.2	27.9	2.6	2.0
46.5-47.5	11.0	16.3	29.5	1.9	1.3
47.5-48.5	11.5	14.5	31.1	1.3	0.4
48.5-49.5	12.0	12.8	32.7	0.9	0.11
49.5-50.5	12.6	11.3	34.4	0.6	0.11
50.5-51.5	13.1	9.9	36.2	0.4	0.00
51.5-52.5	13.6	8.6	38.0	0.3	
52.5-53.5	14.2	7.4	39.8	0.2	
53.5-54.5	14.7	6.3	41.6	0.1	
54.5-55.5	15.3	5.4	43.6	0.1	
55.5-56.5	15.9	4.6		0.0	
56.5-57.5	16.5	3.9			
57.5-58.5	17.1	3.3			
58.5-59.5	17.7	2.7			
59.5-60.5	18.3	2.2			
60.5-61.5	19.0	1.8			
61.5-62.5	19.6	1.5			
62.5-63.5	20.3	1.2			
63.5-64.5	20.9	1.0			
64.5-65.5	21.6	0.8			
65.5-66.5	22.3	0.6			
66.5-67.5	23.0	0.5			
67.5-68.5	23.7	0.4			
68.5-69.5	24.5	0.3			

Table 26. Continued

	54-70	
Age Interval in years	Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval
69.5-70.5	25.2	0.2
70.5-71.5	26.0	0.2
71.5-72.5	26.7	0.1
72.5-73.5	27.5	0.1
73.5-74.5		0.0

Table 27. Retirement Rate Results for Stubs 60-70 and 60-40

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Age Interval in years	Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	Retirement Rate for interval in per cent	Per cent Surviving at beginning of interval	Actual Per cent Surviving at beginning of interval
0.0- 0.5		100.0		100.0	100.0
0.5- 1.5	0.3	100.0	0.8	100.0	100.0
1.5- 2.5	1.6	99.7	1.9	99.2	100.0
2.5- 3.5	3.1	98.1	3.2	97.3	97.3
3.5- 4.5	4.8	95.1	4.5	94.2	94.6
4.5- 5.5	6.6	90.5	6.0	90.0	91.9
5.5- 6.5	8.6	84.5	7.4	84.6	86.5
6.5- 7.5	10.8	77.2	9.0	78.3	73.0
7.5- 8.5	13.2	68.9	10.6	71.3	70.3
8.5- 9.5	15.7	59.8	12.3	63.7	59.5
9.5-10.5	18.5	50.4	14.1	55.9	59.5
10.5-11.5	21.4	41.1	15.9	48.0	45.9
11.5-12.5	24.5	32.3	17.9	40.4	40.5
12.5-13.5	27.7	24.4	19.8	33.2	32.5
13.5-14.5	31.2	17.6	21.9	26.6	29.7
14.5-15.5	34.8	12.1	24.0	20.8	21.8
15.5-16.5	38.6	7.9	26.3	15.8	10.8
16.5-17.5	42.6	4.9	28.5	11.6	8.1
17.5-18.5	46.7	2.8	30.9	8.3	8.1
18.5-19.5	51.1	1.5	33.3	5.7	5.4
19.5-20.5	55.6	0.7	35.8	3.7	2.7
20.5-21.5	60.3	0.3	38.4	2.4	2.7
21.5-22.5	65.1	0.1	41.0	1.5	2.7
22.5-23.5		0.0	43.8	0.9	0.0
23.5-24.5			46.6	0.5	
24.5-25.5			49.4	0.3	
25.5-26.5			52.4	0.2	
26.5-27.5			55.4	0.1	
27.5-28.5				0.0	

CHAPTER VI

CONCLUSIONS

Determination of Average Life and Goodness of Fit.--Before determining average life and goodness of fit, logic was applied to the Gompertz-Makeham results. This meant reducing all survival values above 100 per cent to 100 per cent. Also, survival data which began at a figure less than 100 per cent, rose, and then declined was corrected so that all earlier figures were as large as the maximum point reached. Logic also dictates that all results specify 100 per cent surviving at zero age.

The average life for a table of survivor data is found by adding the column of survival values, having reduced the first value from 100 to 50 per cent. This constitutes a form of numerical integration. The sum is divided by 100 and the result is the average service life. The results for the actual data and for the extensions is shown in Table 28. The sum of the squared deviations divided by the number of points involved was chosen as a measure of goodness of fit. The summation was stopped when the actual data reached zero surviving. The sums of the squared deviations and the number of points involved are shown separately for the length of the stub and for the extension in Table 29.

For the purpose of comparison, the average life figures were converted into errors as a per cent of actual average life. This data is shown in Table 30. The sum of the squared deviations for the stubs divided by the number of points in the stub are shown in Table 31. Similar data for the extensions only appear in Table 32.

Table 28. Average Service Lives for the Actual Data and Extended Stubs

Stub	Actual	Gompertz-Makeham	Retirement Rate	Iowa Type Curves
9-70	9.2		6.3	8
9-40	9.2	7.6		8
29-70	12.8	10.9	13.7	13
29-40	12.8	12.8	12.8	13
33-70	25.9	24.3	24.6	25
33-40	25.9		25.3	25
43-70	9.8		9.7	9
43-40	9.8		9.8	9
54-70	28.8	31.6	32.8	28
54-40	28.8	29.1	28.8	28
60-70	10.9	7.4	10.2	10
60-40	10.9		11.0	10

Note: All lives in years except 29 which is in 100 hours.

Table 29. Sums of Squared Deviations and Number of Points for Stubs and Extensions

Method and Stub Used	Stub Only		Extension Only	
	Sum of Squared Deviations	N_s	Sum of Squared Deviations	N_e
Gompertz-Makeham				
9-40	137.75	9	2261.30	17
29-70	2172.09	12	4832.25	6
29-40	85.05	14	57.74	4
33-70	0.83	21	1287.48	29
54-70	24.48	26	4199.72	25
54-40	78.68	33	171.76	18
60-70	866.70	8	12936.68	15
Retirement Rate				
9-70	12.90	5	7589.14	21
29-70	87.07	12	1077.69	6
29-40	400.15	14	5.31	4
33-70	2.51	21	860.14	29
33-40	76.74	27	50.55	23
43-70	111.33	9	37.73	5
43-40	140.23	10	56.74	4
54-70	18.68	26	6309.48	25
54-40	258.41	33	69.08	18
60-70	26.54	8	552.31	15
60-40	72.13	12	51.61	11
Iowa Type Curves				
9-70	107.22	5	373.19	21
9-40	146.45	9	333.96	17
29-70	36.44	12	859.03	6
29-40	295.95	14	739.82	4
33-70	22.60	21	143.80	29
33-40	100.80	27	65.60	23
43-70	30.42	9	381.32	5
43-40	96.03	10	315.71	4
54-70	155.53	26	283.32	25
54-40	350.04	33	88.81	18
60-70	46.61	8	318.56	15
60-40	102.19	12	262.98	11

Note: Squared deviations are in per cent squared or 10^{-4}.

Table 30. Per Cent Error of Average Service Lives of Extended Stubs

Stub Used	Gompertz-Makeham	Retirement Rate	Iowa Type Curves	Actual Average Service Life in years
9-70		31.5	13.0	9.2
9-40	17.4		13.0	9.2
29-70	14.8	7.0	1.6	12.8 *
29-40	0.0	0.0	1.6	12.8 *
33-70	6.2	5.0	3.5	25.9
33-40		2.3	3.5	25.9
43-70		1.0	8.2	9.8
43-40		0.0	8.2	9.8
54-70	9.7	13.9	2.8	28.8
54-40	1.0	0.0	2.8	28.8
60-70	32.1	6.4	8.2	10.9
60-40		0.9	8.2	10.9

* Units of 100 hours

Table 31. Sums of Squared Deviations Divided by Number of Points for Stubs Only

Stub	Gompertz-Makeham	Retirement Rate	Iowa Type Curves
9-70		2.6	21.4
9-40	15.3		16.3
29-70	181.0	7.3	3.0
29-40	6.1	28.6	21.2
33-70	0.0	0.1	1.1
33-40		2.8	3.7
43-70		12.4	3.4
43-40		14.0	9.6
54-70	0.9	0.7	6.0
54-40	2.4	7.8	10.6
60-70	108.3	3.3	5.8
60-40		6.0	8.5

Note: Figures have been multiplied by 10,000.

Table 32. Sums of Squared Deviations Divided by

Number of Points for Extension Only

Stub Used	Gompertz-Makeham	Retirement Rate	Iowa Type Curves
9-70		361.4	17.8
9-40	133.0		19.6
29-70	805.4	179.6	143.2
29-40	14.4	1.3	185.0
33-70	44.4	29.7	5.0
33-40		2.2	2.8
43-70		7.5	76.3
43-40		14.2	78.9
54-70	168.0	252.4	11.3
54-40	9.5	3.8	4.9
60-70	862.4	36.8	21.2
60-40		4.7	23.9

Note: Figures have been multiplied by 10,000.

Discussion of Results.--First let us compare Tables 31 and 32 to see if there is any relation between closeness of fit to the stub and good forecasting. For the Gompertz-Makeham applications goodness of extended fit seems to follow goodness of stub fit. This may be easily seen in poor values for 29-70 and 60-70, and relatively good values for 29-40 and 54-40 in both tables. For the Retirement rate applications, this relationship seems to be reversed. 9-70, 29-70, 33-70, 54-70, and 60-70 have rather good fits with the stub, but relatively poor fits in their extensions. On the other hand, 29-40 gives a good fit with the latter data, but a poor fit with the stub. The results for the Iowa Method applications do not definitely show the direct or indirect relationship. These observations might be explained by the fact that the Gompertz-Makeham formula is a rather inflexible representation and will give good results for the stub when the data follows the proper form. In these cases, the latter data seems also to follow the Gompertz-Makeham form. The Retirement Rate method is such as to enable it to give a fairly good fit to any stub, yet this very closeness of fit to the stub may cause large error in forecasting.

A comparison of Tables 30 and 32 will indicate whether in any cases a poor prediction as to the form of the extension has by chance resulted in a relatively good estimate of average life. Also, we shall find which figures can be most advantageously studied for the effect of other variables. The performance of Gompertz-Makeham in regard to average life and predicted shape is relatively the same. This holds true for the Retirement Rate results, except in the case of 43-40 where the prediction as to average life is excellent, but the fit of the extension is only fair. The Iowa Method results show fairly good agreement, except for Property Groups 9 and 29. In the case of 9, the average life estimate erred 1.2 years, a large percentage of the actual life of 9.2 years. For Group 29 the average life estimate is good, but the fit of the extension is quite poor. Therefore, since there is relatively good agreement between efficiency in regard to average life and goodness of fit of the extension, and the goodness of fit table is more sensitive, Table 32 will be used for further consideration of the variables.

The results tabulated in Table 32 will be considered in regard to the effect of method, length of stub, shape of curve, size of property group, and number of points available in the stub.

It is obvious from the results that the performance of the Gompertz-Makeham formula was inferior to the other two methods. In no case does it give the best fit. It results in four poor fits out of the seven cases where it could be applied. Comparing the Retirement Rate and Iowa Method results for five of the shorter stubs, the Iowa Method gives better results. Three of the Retirement Rate results for the short stubs are quite large. The Retirement Rate method gives better results for five of the longer stubs. Thus, these two methods have both proved effective, but adaptable to different conditions.

It should be pointed out that the increase in several of the Iowa figures from the short to the long stub is due to the use of the same extension for both (explained in Chapter IV) and a greater proportionate decrease in N_e than the sum of the squared deviations for the longer stub. The Iowa Method shows its poorest results for Group 29. The data is for incandescent lamps and the survivor curve drops off very sharply in the later periods. Type curves which are intended to represent industrial or utility property might not be expected to handle a case where the variability of the lives of the units is so small. The close control of the materials and processes of manufacture, as well as the identity of the conditions within two bulbs while burning, result in a similarity of life spans for different units such as would seldom be found with industrial equipment.

In regard to length of stub, Gompertz-Makeham failed completely on three long stubs and two short ones. In the two cases where both stubs gave results, the long stubs were better. In four cases, the Retirement Rate method gave much better results for the longer stub. The effect of stub length on the Iowa Method is very small, as discussed in the last paragraph and in Chapter IV.

Table 33 was compiled to reveal any major effects of shape, size of property group, or number of points available in the stub. The performance of a method upon the several property groups was rated according to its best performance on either of the two stubs. Then the performances for the several property groups were ranked in order of increasing magnitude--best to worst. The code numbers signifying the final Iowa shape classification for the data are shown below the group code numbers in the table. In regard to size of property group, there is little distinction except for 29 and 60 which represent respectively 75 and 37 units.

These are marked "small" in the table. The number of points available in the stub which gave the best fit for the group is listed under the group code number.

In studying Table 33, the relatively small amount of data and the fact that several characteristics are associated with each property group prevents much generalization. For example, is the failure of Gompertz-Makeham with Property Group 60 due to the data being symmetrical, the group being small, a small number of points in the stub, other variables, or a combination of variables? However, some inferences may perhaps be drawn.

Considering all three methods, good results were usually obtained with Groups 33, 54 and 29. Group 29 had only 75 units. Groups 33 and 54 had the largest number of points in their stubs. These curves have a similar degree of rather high peakedness. The rather poor results with Groups 43, 60 and 9 might indicate the effects of fewer points in their stubs.

Considering each method in turn, Gompertz-Makeham gave really satisfactory results for Groups 54 and 29 only. These are both right modal types. This corresponds with the observations of Winfrey (33). The Retirement Rate Method gives its best results where the property group is small. Its effectiveness is little influenced by shape. There is an indication that in these tests, the Iowa method gave poorer results with the smaller groups.

Finally, a brief comment as to relative ease of application of the three methods. Application of the Gompertz-Makeham Formula and extension of the Retirement Rate data both require lengthy and complex computation in finding preliminary constants and in the tabulation of values. Application of Iowa Type Curves was somewhat easier with the method used here, and would be much easier where many curves were to be extended and the proper preliminary graphs were prepared. However, similar tabulated data and prepared graphs might be used for either of the mathematical methods, or typical data based on company experience, if the volume of work was large.

Table 33. Relationships Between Shape, Size of Property Group, Number of Points in Stub, and Goodness of Fit

	Best Fit					Worst Fit
<u>Gompertz-Makeham</u>						
Property Group	54	29	33	9	60	43
Shape	R_3	R_4	L_3	L_1	S_1	S_4
Size of Group		Small			Small	
Number of Points in Stub	33	14	21	9	8	10
<u>Retirement Rate</u>						
Property Group	29	33	54	60	43	9
Shape	R_4	L_3	R_3	S_1	S_4	L_1
Size of Group	Small			Small		
Number of Points in Stub	14	27	33	12	9	5
<u>Iowa Type Curves</u>						
Property Group	33	54	9	60	43	29
Shape	L_3	R_3	L_1	S_1	S_4	R_4
Size of Group				Small		Small
Number of Points in Stub	27	33	5	8	9	12

Conclusions.--

1. In this study, the Gompertz-Makeham Formula proved quite unsatisfactory for the extension of stub survivor curves of physical property.

2. The Retirement Rate Method can be expected to give quite accurate results for extending stub survivor curves when 40 per cent or less of the property group is surviving.

3. The use of Iowa Type Curves for the extension of stub survivor curves gives reasonably satisfactory results when 70 per cent or less of the property group is surviving, with relatively small labor of application.

Areas Needing Further Study.--In the course of this investigation, several problems were touched upon which seem of interest and importance, but have apparently received little study.

First, this thesis is an empiric investigation of the effectiveness of several methods of extending stub survivor curves. Fairly good predictions are obtained. However, some writers have pointed out that different parts of a particular survivor curve may be best represented by different type curves or mathematical formulations with different constants. A study of interest would concern the possible difference between the best representation for the first and last halves of a number of survivor curves.

Second, no mathematical justification or theorizing was found in the literature concerning the Retirement Rate Method. The method worked quite successfully in this study and is widely advocated. Mathematical justification exists for the Gompertz-Makeham Formula and several other mathematical models.

Third, a mathematical approach to survivor curves including as many of the practical variables as possible seems needed. At present there seems to be a rather large gulf between the mathematical theorists and the people who actually handle survivor data. When such variables as maintenance, expected intensity of usage, chance of destruction, pattern of innovation, and others, can be appropriately mathematically expressed--before the property is put into use--statistical study of property survival will have reached a position of great value to industry.

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